

Probing statistical isotropy of cosmological radio sources using SKA

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Abstract: There currently exist many observations which are not consistent with the cosmological principle. We review these observations with a particular emphasis on those relevant for Square Kilometre Array (SKA). In particular, several different data sets indicate a preferred direction pointing approximately towards the Virgo cluster. We also observe a hemispherical anisotropy in the Cosmic Microwave Background Radiation (CMBR) temperature fluctuations. Although these inconsistencies may be attributed to systematic effects, there remains the possibility that they indicate new physics and various theories have been proposed to explain them. One possibility, which we discuss in this review, is the generation of perturbation modes during the early pre-inflationary epoch, when the Universe may not obey the cosmological principle. Better measurements will provide better constraints on these theories. In particular, we propose measurement of the dipole in number counts, sky brightness, polarized flux and polarization orientations of radio sources. We also suggest test of alignment of linear polarizations of sources as a function of their relative separation. Finally we propose measurement of hemispherical anisotropy or equivalently dipole modulation in radio sources.

Keywords: SKA – Cosmological Principal – Kinematic Dipole – Intrinsic Dipole

1 Introduction

The Big Bang model is based on the cosmological principle which states that the Universe is isotropic and homogeneous, i.e. there is no preferred direction or position. It is essentially an assumption and cannot be proven on the basis of the symmetries of the fundamental action. In particular, it applies only in a statistical sense, after averaging over distances of order 100 Mpc. Furthermore there is a preferred frame of reference, the so called cosmic frame of rest. The Universe appears isotropic and homogeneous only in this frame. Within the Big Bang paradigm, the Universe may not be isotropic and homogeneous at very early times. It acquires this property during inflation. It has been explicitly shown that starting from a wide range of anisotropic but homogeneous Bianchi models, the Universe quickly becomes isotropic during inflation (Wald, 1983). However other models also exist which do not obey this principle. In this article we review the current status of the tests of the cosmological principle. We also review some of the theoretical attempts to explain the observed violations of this principle.

Observationally it is easier to test isotropy in contrast to homogeneity because it requires only angular positions of the sources. A test of homogeneity requires three dimensional mapping of the Universe. Here we shall primarily be interested in observations which test isotropy. However we point out that an observed violation of isotropy may arise in a fundamental model which may be anisotropic or inhomogeneous or both.

Even within the Big Bang model, the Universe is not strictly isotropic and homogeneous. It obeys this property only in a statistical sense in the cosmic frame of rest. For example, let us

consider the matter density $\rho(t, \mathbf{x})$ where t is the cosmic time and \mathbf{x} the comoving coordinate. Its spatial distribution can be expressed as,

$$\rho(t, \mathbf{x}) = \rho_0(t) + \delta\rho(t, \mathbf{x}). \quad (1)$$

Here $\rho_0(t)$ is the mean density and $\delta\rho$ the fluctuations, such that

$$\langle \delta\rho(t, \mathbf{x}) \rangle = 0. \quad (2)$$

Here the angular brackets represent ensemble average. An estimate of this mean is obtained by averaging $\delta\rho$ over a sufficiently large patch of the Universe. We expect this distance scale to be of order 100 Mpc. At smaller scales the matter density shows considerable clustering and the cosmological principle does not apply. Statistical isotropy (SI) and homogeneity implies

$$\langle \delta\rho(t, \mathbf{x})\delta\rho(t, \mathbf{x}') \rangle = f(|\mathbf{x} - \mathbf{x}'|), \quad (3)$$

i.e., the two point correlations depend only on the distance between the two points and not on the direction or the position. If we relax the assumption of isotropy then these correlations can also depend on the direction of the vector $\mathbf{x} - \mathbf{x}'$. If we also allow inhomogeneity, then we can also get dependence on the mean position $(\mathbf{x} + \mathbf{x}')/2$. As we have mentioned above, statistical isotropy applies only in the cosmic frame of rest. If we are in motion with respect to this frame with velocity \mathbf{v} , then at leading order in $|\mathbf{v}|$, the matter distribution is expected to show a dipole distribution peaked in the direction of \mathbf{v} .

Within the Big Bang model the CMBR temperature field can be decomposed as

$$T(\hat{n}) = T_0 + T_1 \hat{\lambda} \cdot \hat{n} + \Delta T(\hat{n}) \quad (4)$$

where \hat{n} is a unit vector in the direction of observation, T_0 the mean temperature, T_1 the amplitude of the CMBR dipole, $\hat{\lambda}$ the dipole axis and ΔT the primordial fluctuations in temperature. Here the dipole contains both the kinematic contribution, arising due to local motion, as well as the contribution due to primordial fluctuations. Hence ΔT contains only multipoles corresponding to $l \geq 2$, i.e., quadrupole and higher. We use the spherical polar coordinates (θ, ϕ) to label the direction of observation. As in the case of density fluctuations, we have

$$\langle \Delta T(\hat{n}) \rangle = 0. \quad (5)$$

Observationally, $T_0 \approx 2.73\text{K}$, $T_1/T_0 \sim 10^{-3}$ and $\Delta T/T_0 \sim 10^{-5}$. Statistical isotropy implies that the two point correlation function satisfies

$$\langle \Delta T(\hat{n}_i) \Delta T(\hat{n}_j) \rangle = C(\hat{n}_i \cdot \hat{n}_j), \quad (6)$$

i.e., it is a function only of the angle between the two observation points \hat{n}_i and \hat{n}_j . It is useful to expand the temperature fluctuations in terms of the spherical harmonics. We obtain

$$\frac{\Delta T(\hat{n})}{T_0} = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \quad (7)$$

where a_{lm} are the coefficients of this expansion. These also satisfy $\langle a_{lm} \rangle = 0$. Furthermore statistical isotropy implies that

$$\langle a_{lm} a_{l'm'}^* \rangle_{\text{iso}} = C_l \delta_{ll'} \delta_{mm'}, \quad (8)$$

where C_l is the standard CMBR power.

The cosmological principle is supported by the Cosmic Microwave Background Radiation (CMBR) and galaxy surveys. The observed CMBR temperature $T(\theta, \phi)$ is found to be isotropic

to a very good approximation. As mentioned above, the largest deviation from isotropy arises due to dipole which is of order 10^{-3} . The dominant dipole contribution arises due to the velocity of the solar system (\mathbf{v}_{CMB}) relative to the cosmic frame of rest. Its magnitude \mathbf{v}_{CMB} and direction (l, b) in galactic coordinates are respectively found to be 369 ± 0.9 Km/s and $(263.99^\circ \pm 0.14^\circ, b = 48.26^\circ \pm 0.03^\circ)$ (Kogut et al., 1993; Hinshaw et al., 2009). The number density and brightness of distant radio galaxies are also observed to be isotropic to a good approximation. However there are many observations which suggest a potential violation of the cosmological principle. In particular the local velocity $\mathbf{v}_{\text{radio}}$ extracted from the observed dipole in the number density and brightness of radio sources is not found to be in agreement with \mathbf{v}_{CMB} . The direction agrees but the magnitude is found to be approximately three times larger. We review such observed violations of the cosmological principle in the next section. In section 3 we shall present a theoretical model which may potentially explain these observations. In section 4 we shall discuss tests of statistical isotropy at Square Km Array (SKA) and will conclude in section 5.

2 Observed violations of statistical isotropy

The assumption of statistical isotropy is built into the Inflationary Big Bang model, which is the Standard Model of Cosmology. The predictions of the standard model agree remarkably well with observations which is a real success for the modern era of precision cosmology. Despite the success of the theory there are tantalizing evidences which highlight small but persistent departures from the predictions of the isotropic theory. Such observations are mostly in the large distance scale observations. In this section we will discuss some of the observed violations in statistical isotropy found in different observations with a particular emphasis on those relevant for SKA.

2.1 Kinematic Dipole

Before we discuss the major observations of SI violation, it is important to understand that the Cosmological Principle is valid only in the cosmic frame of rest. The Earth is not at rest with respect to this frame. It is rotating about the Sun, which in turn is rotating about the centre of the Milky Way; the Milky Way moves with respect to the Local Group barycenter, which in turn moves about the large scale structures around it. The combined motion due to these peculiar velocities ensures that our frame of observation has a relative velocity with respect to the cosmic frame of rest. This leads to a dipole in the observer frame even if the field is isotropic in the cosmic rest frame. This dipole due to Doppler shift of the CMB photons is called the kinematic dipole.

We denote the peculiar velocity of our observation frame by \mathbf{v} and define $\boldsymbol{\beta} = \mathbf{v}/c$. If the temperature field and direction in the cosmic rest frame are identified as T' and \hat{n}' and the unprimed symbols denote the observations in our frame, then

$$T(\hat{n}) = \frac{T'(n')}{\gamma(1 - \hat{n} \cdot \boldsymbol{\beta})}, \quad (9)$$

and

$$\hat{n} = \frac{\hat{n}' + [(\gamma - 1)\hat{n}' \cdot \hat{v} + \gamma\boldsymbol{\beta}]\hat{v}}{\gamma(1 + \hat{n}' \cdot \boldsymbol{\beta})} \quad (10)$$

where $\gamma = \sqrt{1 - \beta^2}$. Due to Doppler shifting the intensity distribution of the CMB photons gets modified. We measure $T(\hat{n})$ and use these relations to obtain the temperature field in the cosmic rest frame along with the peculiar velocity of the observation frame.

The large scale structures also acquire a dipole due to Doppler and aberration effects caused by our local motion. The flux density of radio sources typically shows a power law dependence

on frequency. Furthermore the number density of sources depends on the flux density. Most large scale structure surveys operate in limited frequency ranges and have a lower limit on the flux density. Due to Doppler effect, the frequencies in the direction of motion of the frame are blueshifted and are redshifted in the opposite direction. Due to this effect and the intensity cuts on the survey, sources will shift in and out of the range of observations. Hence in the direction of motion more objects are blueshifted into the observation frequencies while in the other hemisphere more sources are redshifted out of the range. Combining the two effects — the Doppler shift leads to a small dipole in a limited frequency and intensity range large scale structure survey (Ellis and Baldwin, 1984; Tiwari et al., 2015).

The motion of the reference frame also leads to the aberration effect. This produces a shift in the angular position of the source. Thus the apparent positions of an isotropic distribution of sources get shifted towards the direction of motion of the frame, creating a dipole. This effect is of the same order as $\beta \sim 10^{-3}$ and is relevant for large scale structure dipole studies. Combined effect of the Doppler shift and aberration produces the *kinematic dipole*.

2.2 Observed Dipole in Large Scale Structures

Large scale structures are essentially objects formed by non-linear physics. When observed on small survey volumes the non-linear physics produces structures that would deviate from isotropy and homogeneity. Thus the local non-linear components of a survey would produce a *local structure dipole*. This is not a violation of SI, because the Cosmological Principle is not valid on this scale. It is only when a very large survey volume, of length scales greater than a few hundred Mpc, is considered that the Cosmological Principle is applicable and can be tested for SI violations. If a dipole component is present over and above the local structure and the kinematic dipole, then it is of cosmological origin and is called the *intrinsic dipole*. We are essentially interested in the intrinsic component in SI violation study.

The most significant study of dipoles in the large scale structure has been done with the NRAO VLA Sky Survey (NVSS) radio catalogue containing 1773484 radio sources (Condon et al., 1998). The survey’s operating frequency is 1.4 GHz and covers the entire northern hemisphere above a declination of -40° and has a mean redshift ~ 1 . For radio sources, in the cosmic rest frame, the flux density S follows a power law relation with frequency ν , $S \propto \nu^{-\alpha}$, with $\alpha \approx 0.75$. The differential number count of radio sources per unit solid angle per unit flux density follows the power law: $n(\theta, \varphi, S) \propto S^{-1-x}$, where the spectral index x is close to unity. Due to the kinematic effects discussed above, it is clear that both the number counts and sky brightness would show a dipole. We denote these by $\mathbf{D}_N^{\text{kin}}$ and $\mathbf{D}_S^{\text{kin}}$ respectively. These kinematic dipoles are given by

$$\mathbf{D}_S^{\text{kin}} = [2 + x(1 + \alpha)]\beta, \quad \mathbf{D}_N^{\text{kin}} = [2 + x(1 + \alpha)]\beta, \quad (11)$$

i.e. both are described by the same formula (Ellis and Baldwin, 1984; Tiwari et al., 2015; Singal, 2011). Since the velocity of our observation frame with respect to the cosmic rest frame is already known from CMB experiments, we can make a prediction for the kinematic dipole.

The earliest attempt to extract the NVSS dipole was made by Blake and Wall (2002), where they claimed to find the dipole amplitude approximately two sigmas larger than the expected kinematic dipole. The extracted direction, however, showed good agreement with expectations. This was revisited later by several authors, who found an even larger deviation from the amplitude of the kinematic dipole. These results are summarised in Table 1.

We note that the result obtained by Gibelyou and Huterer (2012) shows a much larger deviation from others. Rubart and Schwarz (2013) have shown that the dipole amplitude estimator used by Gibelyou and Huterer is biased. It has a direction bias and as a consequence their dipole direction estimates are not in agreement with other results. The amplitude obtained by Blake and Wall is smaller than that obtained by any of the other authors. Our study of the NVSS dipole (Tiwari et al., 2015) involved studying not just the number count but also

Authors	D_0 ($\times 10^{-2}$)	v ($\times 10^3$ in km/s)	(l, b)
Blake & Wall (2002)	1.05 ± 0.42	0.9 ± 0.3	$(245^\circ, 41^\circ)$
Singal (2012)	1.8 ± 0.3	1.32 ± 0.54	$(239^\circ, 44^\circ)$
Gibelyou and Huterer (2012)	2.7 ± 0.5	1.4 ± 0.3	$(214^\circ, 15^\circ)$
Tiwari et. al. (2015) D_N	1.25 ± 0.40	1.00 ± 0.32	$(261^\circ, 37^\circ)$
Tiwari et. al. (2015) D_S	1.51 ± 0.57	1.21 ± 0.46	$(269^\circ, 43^\circ)$
Rubart and Schwarz. (2013)	1.8 ± 0.6	1.5 ± 0.5	$(239^\circ, 44^\circ)$

Table 1: *NVSS observed dipole amplitude, observation frame peculiar velocity and direction.* Collected results (Blake and Wall, 2002; Singal, 2011; Gibelyou and Huterer, 2012; Tiwari et al., 2015; Rubart and Schwarz, 2013) for the NVSS dipole amplitude and direction with flux densities > 20 mJy (> 15 mJy for Gibelyou and Huterer). Here D_0 is the total observed dipole and v is the peculiar velocity of the observation frame, calculated from D_0 .

the sky brightness dipole. Both observables show similar results with amplitudes exceeding the kinematic dipole predictions by approximately two sigmas. Such excess dipole on such large distance scales suggests a mild signal of potential violation of SI.

The results discussed above while being intriguing need to be reassessed with other data sets due to the limitations of the NVSS catalogue. The NVSS is compiled by use of two different array configurations, one above declination of -10° and one below. This results in systematics in the catalogue. The mean number count becomes a function of declination. Plots of number count density show a large and significant dip below a declination of -15° and a small but linear systematic decrease with increasing right ascension. With a flux cut > 15 mJy, the effect of these systematics can be suppressed to a level where they are no longer visible to the naked eye while plotting. While the work done with the NVSS data try to limit the effect of such systematics, having another deep survey with large sky coverage to test out these results would be very important before we can be sure of SI violation.

The NVSS also contains information about the polarization of the sources. It provides Stokes parameters Q and U for these sources. Using them we can test the isotropy of sources with non-zero polarized flux density P , defined as, $P = \sqrt{Q^2 + U^2}$. The polarized flux density, for radio sources, follows a power law, $P \propto \nu^{-\alpha_P}$, with $\alpha_P \approx 0.75$. The differential number count per unit solid angle, per total flux density S and polarized flux density P is given as $n(\theta, \varphi, P, S) \propto S^{-1-x} P^{-1-x_P}$ in the cosmic rest frame. The kinematic dipole in the number count of significantly polarized sources and the integrated polarized flux density is given by Tiwari and Jain (2015a):

$$\mathbf{D}_{N_P}^{\text{kin}} = [2 + x(1 + \alpha) + x_P(1 + \alpha_P)]\boldsymbol{\beta} \quad (12)$$

$$\mathbf{D}_P^{\text{kin}} = [2 + x(1 + \alpha) + x_P(1 + \alpha_P)]\boldsymbol{\beta} \quad (13)$$

As in the case of Eq. 11, we find that these dipoles also turn out to be identical. The extracted velocities are shown in Table 2. It clearly shows a deviation from the expectations of a kinematic dipole which may indicate the presence of an intrinsic dipole.

Dipole type	D_0 ($\times 10^{-2}$)	v ($\times 10^3$ in km/s)	(l, b)
D_{N_P}	3.3 ± 0.8	2.38 ± 0.61	$(207^\circ, 37^\circ)$
D_P	4.9 ± 1.2	2.87 ± 0.68	$(244^\circ, 20^\circ)$

Table 2: *NVSS dipole amplitude, observation frame peculiar velocity and direction for sources with non-zero polarized flux.* Results from Tiwari and Jain (2015a) with a lower limit on total flux density of 30 mJy and polarized flux density range of $0.1 < P < 100$ mJy.

Study of Sloan Digital Sky Survey (SDSS) by Itoh et al. (2010) also revealed some fascinating

hints of SI violations. The SDSS 6th Data Release photometric catalogue contains over 200 million sources and covers an area of around 8000 deg^2 , with photometric data in five band passes. While the SDSS has a very high fidelity data with low and well understood systematics, its sky coverage is small – at about 20% with a mean redshift ~ 0.3 . This makes the catalogue difficult to use for cosmological purpose. There are also some issues which need to be taken care of in constructing the sample for analysis. The first is to ensure that stars are carefully and reliably removed from analysis. Putting appropriate magnitude range helps in isolating the galaxies. Another well known feature of the SDSS catalogue is the presence of local clustering at large scales. The most well known feature of the SDSS is the *Sloan Great Wall*, at a redshift of ~ 0.08 . Such local clustering has to be removed reliably before the intrinsic dipole can be studied. The expected kinematic dipole amplitude in the SDSS is found to be 1.231×10^{-3} (Itoh et al., 2010). The authors worked with four galaxy samples with different ranges in brightness and photometric redshift. Of them we only discuss two here. These are those samples which are deepest, more relevant from a cosmological point of view. The results we discuss are for the bright deep (BD) and the faint deep (FD) samples. For both, the maximum photometric redshift is ~ 0.9 . The authors performed a χ^2 minimisation with the full covariance matrix. For the BD sample the authors obtained a dipole amplitude of $0.87^{+0.59}_{-0.57} \times 10^{-2}$ along $(l = 290^\circ, b = -10^\circ) \pm 100^\circ$. The FD sample gave a dipole amplitude of $(1.21 \pm 0.23) \times 10^{-2}$ along $(l = 280^\circ, b = 75^\circ) \pm 33^\circ$.

The authors found a dipole excess in all but the BD sample. They suggested that possible contamination in the FD samples from incomplete star-galaxy separation and with incorrectly removed clustering in the data might've caused the large measured dipole in this sample. Another reason for difference between the two samples might be the small sky coverage of the survey. They hoped that a sky survey with a wider coverage would be able to settle the issue.

Yoon et al. (2014) found a dipole in the Wide-field Infrared Survey Explorer-Two Micron All Sky Survey (WISE-2MASS) catalogues. The WISE catalogue has 757 million sources which are however uncategorised. The authors use the 2MASS catalogue with joint intensity limits to select data for analysis. The GAMA D2 data was used to model the redshift distribution for the WISE catalogue. The selected object field is shallow with mean redshift of 0.139 and goes up to a maximum of 0.4. They follow a method similar to that of Gibelyou and Huterer (2012) to estimate the dipole. With a 20° galactic plane cut, the result they obtained was $(5.2 \pm 0.2) \times 10^{-2}$ along $(l = 308^\circ \pm 4^\circ, b = -14^\circ \pm 2^\circ)$, which exceeded the theoretical expectations from local structure dipole. The theoretical dipole amplitude expected being 2.3 ± 1.2 . They did not consider the effect of the kinematic dipole which has an order of magnitude lesser contribution and could not be sufficiently tested with the shallow data.

In the last few years the tests of SI violations with large scale structures have gathered steam. With deeper data and with greater sky coverage, better constraints can be put on SI violations and thereby constraining SI violating model parameters and mechanisms. With improvement in data fidelity and understanding of systematics, we may be able to reduce these errors and find out if truly these SI violations are consistent.

2.3 Virgo Alignment

A very curious feature of SI violations is the alignment of various preferred directions in different data sets. Several observations at wide range of frequencies suggest a preferred direction pointing roughly towards the Virgo supercluster, which is close to the direction of the observed CMBR dipole. We have already discussed the possible presence of intrinsic dipole in the number counts, sky brightness as well as polarized radio flux. Furthermore, the CMB quadrupole, CMB octopole, radio and optical polarizations from distant sources also indicate a preferred direction pointing roughly towards Virgo. Next we briefly describe each of these effects.

The distribution of polarization angles of distant radio galaxies indicates a dipole pattern. Here the observable is $\beta = \chi - \phi$, where χ is the linear polarization angle and ϕ is the orientation

angle of the galaxy. This parameter shows a dipole distribution across the sky. The significance of the effect is found to be 3.5σ after making a cut which eliminates the central peak in the distribution of the rotation measures (RM) (Jain and Ralston, 1999; Jain and Sarala, 2006). The preferred direction of the dipole is found to be $l = 259^\circ$, $b = 62^\circ$ in galactic coordinates.

The CMBR quadrupole and the octopole, i.e. multipoles corresponding to $l = 2, 3$, also indicate a preferred direction $((l, b) \sim (250^\circ, 60^\circ))$, pointing roughly towards Virgo. Statistical isotropy would imply that these are independent of one another as well of other multipoles, such as the dipole. However the preferred axis of both these multipoles points approximately in the direction of the CMB dipole (de Oliveira-Costa et al., 2004; Ralston and Jain, 2004). This is rather surprising! Furthermore, it is difficult to explain this alignment in terms of bias or foreground effects (Aluri et al., 2011). The procedure for extraction of the preferred direction has been developed in (de Oliveira-Costa et al., 2004; Ralston and Jain, 2004; Samal et al., 2008). There also exist other methods for testing statistical isotropy of CMBR (Hajian et al., 2005; Copi et al., 2007). One may either maximize the angular momentum dispersion $\langle \frac{\delta T}{T} | (\hat{n} \cdot \hat{L})^2 | \frac{\delta T}{T} \rangle$ (de Oliveira-Costa et al., 2004; Bennett et al., 2011). Alternatively one may calculate the principle eigenvector of the power tensor for the two modes (Ralston and Jain, 2004; Samal et al., 2008, 2009). For $l = 2, 3$ it has a simple interpretation. Both these multipoles appear to be planar, i.e., all the hot and cold spots lie roughly in the plane. The direction perpendicular to this is the preferred axis. In more detail, one finds that most of the contribution to the octopole power comes from $|m| = 3$ coefficients. When maximized over direction the $|a_{3,3}|^2$ and $|a_{3,-3}|^2$ contribute approximately 94% of the total power in the octopole (Bennett et al., 2011). This unusual planar power distribution in octopole is another CMB anomaly at large length scales.

The optical polarizations from distant quasars show an alignment over very large distance scales (Hutsemekers, 1998; Jain et al., 2004), i.e. the linear polarizations of different sources are observed to point in the same direction. A very strong alignment effect is seen in the direction of Virgo as well as in the diametrically opposite direction. The angular dependence of the two point correlations of these polarizations was studied in Ralston and Jain (2004). This dependence was not found to be statistically significant. However it is interesting that the correlations were found to maximize along an axis pointing towards Virgo (Ralston and Jain, 2004). Hence we see that a wide range of phenomenon, ranging from radio number densities, sky brightness, polarized flux, polarization angles, CMBR dipole, quadrupole and octopole as well as the optical polarizations from quasars indicate a preferred direction pointing approximately towards Virgo. Below we mention one more effect related to CMBR which also indicates this direction.

2.4 Dipole Modulation in CMBR

The present era of precision cosmology was ushered in by the precision measurements of the cosmic microwave background (CMB), so our most important indicators of SI violations have come from the CMB observations. Of the various departures from SI predictions, the dipole modulation of the CMB temperature fluctuation field is the most important. The original claims were made by Hansen et al. (2004), reporting a hemispherical power asymmetry in the CMB temperature observations made by the Wilkinson Microwave Anisotropy Probe (WMAP). The authors masked the galactic plane in the CMB temperature maps and analysed the binned angular power spectrum on circular patches of varying sizes, oriented about different directions in the sky. They reported significantly different C_ℓ 's in the northern and southern galactic hemispheres for the multipole range $2 - 40$. The $2 - 4$ range was reported to have contribution from the galactic foreground residuals and the signal being directional along the galactic poles. The power spectrum estimates in $5 - 40$ range however showed asymmetry levels which could not be justified by systematics and noise. The asymmetry in $5 - 40$ range was found to maximize along $(57^\circ, 10^\circ)$ in Galactic coordinates, which is close to the ecliptic axis. In the frame of maximum asymmetry, they found that all the $5 - 40$ multipoles in the northern hemisphere

have less power than the average amplitudes, while in the southern hemisphere most of the multipoles in the range have more power than the average amplitude. The authors also claimed a similar signal of lower significance in the COsmic Background Explorer (COBE) data thereby ruling out systematics as a possible source of the signal.

Gordon (2007) proposed a model of linear modulation of the isotropic temperature fluctuation field to phenomenologically represent hemispherical anisotropy. In this model, the temperature fluctuation (δT) observed along a direction \hat{n} , is given by

$$\delta T(\hat{n}) = \delta T_{\text{iso}}(\hat{n}) [1 + f(\hat{n})], \quad (14)$$

where $f(\hat{n})$ is a direction dependent function that modulates δT_{iso} , the isotropic temperature fluctuation field¹. The modulating function $f(\hat{n})$ is assumed as $A\hat{\lambda} \cdot \hat{n}$. This linear modulation along a preferred direction $\hat{\lambda}$ and with amplitude A , would result in a dipole modulation at the surface of last scattering. However, it is important to understand that hemispherical power asymmetry is not the same as dipole modulation. A dipole modulation model will naturally give rise to hemispherical asymmetry but hemispherical power asymmetry does not necessitate a dipole modulation.

In 2009, following the release of WMAP five-year data, Hoftuft et al. (2009) estimated the three parameters A , and two components of $\hat{\lambda}$ from the data, maximizing the log-likelihood for the dipole modulation model. The observed data along a direction (\hat{n}) is written as in (14) but with an additive noise term to read $d(\hat{n}) = \delta T(\hat{n}) + N(\hat{n})$. The signal covariance matrix for such a model is given by (Hoftuft et al., 2009)

$$\mathbf{S}_{\text{mod}}(\hat{n}, \hat{m}) = \left[1 + A\hat{\lambda} \cdot \hat{n}\right] \mathbf{S}_{\text{iso}}(\hat{n}, \hat{m}) \left[1 + A\hat{\lambda} \cdot \hat{m}\right]. \quad (15)$$

The isotropic signal covariance matrix \mathbf{S}_{iso} is written as

$$\mathbf{S}_{\text{iso}}(\hat{n}, \hat{m}) = \frac{1}{4\pi} \sum_i (2\ell + 1) C_\ell P_\ell(\hat{n} \cdot \hat{m}). \quad (16)$$

Here the P_ℓ s are the Legendre polynomials. The full covariance matrix then reads (Hoftuft et al., 2009)

$$\mathbf{C} = \mathbf{S}_{\text{mod}}(A, \hat{\lambda}) + \mathbf{S}_{\text{iso}} + \mathbf{N} + \mathbf{F}, \quad (17)$$

with \mathbf{N} and \mathbf{F} as noise covariance and foregrounds respectively. Assuming the signal and noise both to be Gaussian the log-likelihood takes the form (Hoftuft et al., 2009):

$$-2\ln\mathcal{L}(A, \hat{\lambda}) = \mathbf{d}^T \mathbf{C}^{-1} \mathbf{d} + \ln|\mathbf{C}|. \quad (18)$$

The best-fit results in the $\ell \leq 64$ range, obtained by maximizing the log-likelihood, are given in Table 3. The dipole modulation signal was claimed with a 3.3σ significance for $\ell \leq 64$.

It has been shown (Prunet et al., 2005; Rath and Jain, 2013) that for a dipole modulated temperature fluctuation field given by Eq. 14, with the preferred direction $\hat{\lambda}$ chosen along \hat{z} , the two point correlation function of the spherical harmonic coefficients $a_{\ell m}$ is given by

$$\begin{aligned} \langle a_{\ell m} a_{\ell' m'}^* \rangle &= \langle a_{\ell m} a_{\ell' m'}^* \rangle_{\text{iso}} + \langle a_{\ell m} a_{\ell' m'}^* \rangle_{\text{dm}} \\ &= C_\ell \delta_{\ell\ell'} \delta_{mm'} + A(C_{\ell'} + C_\ell) \times \\ &\quad \left[\sqrt{\frac{(\ell - m + 1)(\ell + m + 1)}{(2\ell + 1)(2\ell + 3)}} \delta_{\ell', \ell+1} + \sqrt{\frac{(\ell - m)(\ell + m)}{(2\ell + 1)(2\ell - 1)}} \delta_{\ell', \ell-1} \right] \delta_{m' m}. \end{aligned} \quad (19)$$

This implies that for a dipole modulated temperature field, the covariance matrix, in spherical harmonic space is not diagonal. The added modulation gives rise to non-zero correlations

¹Note that we have changed the sign in front of $f(\hat{n})$ from ‘-’ to ‘+’ to keep consistency with later work.

Result from	A	(l,b)
Hoftuft et al. (2009) (W5)	0.072 ± 0.022	$(224^\circ, -27^\circ) \pm 24^\circ$
Ade et al. (2014) (P13)	0.065 ± 0.021	$(226^\circ, -17^\circ) \pm 24^\circ$
Ade et al. (2015) (P15)	0.066 ± 0.021	$(225^\circ, -18^\circ) \pm 24^\circ$
Rath et al. (2015) (W9)	0.090 ± 0.029	$(227^\circ, -14^\circ)$
Rath et al. (2015) (P13)	0.074 ± 0.019	$(229^\circ, -16^\circ)$
Ghosh et al. (2016) (P15)	0.078 ± 0.019	$(242^\circ \pm 16^\circ, -17^\circ \pm 20^\circ)$

Table 3: *Best-fit values for the dipole modulation parameters.* W5 and W9 stand for WMAP five-year and nine-year datasets respectively, P13 and P15 stand for Planck 2013 and 2015 SMICA maps.

between ℓ and $\ell \pm 1$ multipoles. So we have studied the dipole modulation feature using this property of non-zero ℓ , $\ell + 1$ correlations by defining a statistic S_H as

$$S_H = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \frac{\ell(\ell+1)}{(2\ell+1)} \sum_{m=-\ell}^{\ell} a_{\ell m} a_{\ell' m'}^* \quad (20)$$

which is a summed estimate of the ℓ , $\ell+1$ correlations in the range $\ell_{\min} \leq \ell \leq \ell_{\max}$. The analysis was performed by setting $\ell_{\min} = 2$ and $\ell_{\max} = 64, 128$ for extraction of different parameters. Some of the results of this analysis are shown in Table 3 and show good agreement with other estimates.

The hemispherical power asymmetry has persisted in the data for three generations of satellite based CMB experiments. The Planck experiment team has tested for both the hemispherical power asymmetry and dipole modulation in their CMB data, finding evidence for both (Ade et al., 2014, 2015). The dipole modulation signal has persisted at $\sim 3\sigma$ level in the 2013 and 2015 data release. The results of the Planck team and the corresponding results with the statistic S_H are shown in Table 3 for comparison.

A test of dipole modulation or equivalently hemispherical anisotropy for the polarization E modes has also been carried out in Ghosh et al. (2016). The low l multipoles of the polarization field are unreliable. Hence the authors only considered multipoles $l \geq 40$. Furthermore they did not test the significance of the effect since it required extensive numerical work in modelling detector noise. Interestingly it was found that the preferred direction in the range $40 \leq l \leq 100$ again points in the direction of Virgo. The direction starts to shift as we extend the upper limit on l . Although the statistical significance of the effect is unknown, it is interesting that the low l multipoles again prefer a direction towards Virgo.

2.5 Dipole Modulation in large Scale Structures

A signal of the dipole modulation has also been investigated in the large scale structures. The first attempt in this direction was made by Hirata (2009) using SDSS quasars. His approach to the problem of searching for dipole modulation in the large scale structures was based on the variation of the amplitude of the linear power spectrum σ_8 . If the CMB hemispherical asymmetry and dipole modulation are of cosmological origins then they should be linked to the primordial curvature perturbations. Such a situation would lead to a gradient in the amplitude of the power spectrum along the preferred direction of the dipole. Since the growth and abundance of large scale structures is very sensitive to the value of σ_8 , the gradient of this parameter can be constrained from the number variations of the large scale structures.

The SDSS quasars were chosen by Hirata to test out the variation of σ_8 . This set had deep distance spread with wide angular coverage. Since these are SDSS objects, the systematics are fairly well understood. One of the drawbacks of the dataset chosen is that the number density of such quasars is small, roughly 1 deg^{-2} . When the preferred direction is fixed along that

obtained by Eriksen et al. (2007) ($l = 225^\circ, b = -27^\circ$), the amplitude of dipole modulation was found as $A = -0.0018 \pm 0.0044$. A search for the best fit direction did not reveal a statistically significant signal. Overall, Hirata's work is strongly indicative of no dipole modulation in the large scale structures.

Fernández-Cobos et al. (2014) searched for the dipole modulation signal in the NVSS. Their approach is a logical extension of the Hoftuft et. al. method, described at the beginning of this section, to the large scale structures, working with the galaxy angular power spectra C_ℓ^{GG} . They worked with three lower flux cuts of 2.5, 5.0 and 10.0 mJy. They corrected for the declination dependent systematics, only for the case of the 2.5 mJy cut, by dividing the entire data map into 70 strips of equal area and rescaling the number density. From their simulation they forecasted a non-negligible dipole modulation with $A = 0.065 \pm 0.013$ along the direction $(l = 224^\circ, -14^\circ) \pm 17^\circ$. However they did not find any evidence of dipole modulation in data. The modulation amplitude A was found to be 0.003 ± 0.015 for 2.5 mJy cut, 0.011 ± 0.016 for 5.0 mJy cut and 0.007 ± 0.014 for 10.0 mJy cut, all of the amplitudes being compatible with null result.

2.6 Alignment of linear polarizations of radio sources

The linear polarizations of radio sources show alignment with one another, analogous to the alignment of optical polarizations from quasars. An alignment on the distance scale of 100 Mpc was reported in Tiwari and Jain (2013) in the JVAS/CLASS sources with polarized flux greater than 1 mJy. This has subsequently been confirmed (Shurtleff, 2014; Pelgrims and Hutsemékers, 2015). An alignment on larger distance scales for the subsample of QSOs in this data set has also been reported in Pelgrims and Hutsemékers (2015). An alignment on the scale of 100 Mpc may be expected within the framework of Big Bang cosmology since sources show correlation with one another on such distance scales. In Tiwari and Jain (2015b) the authors argued that this alignment is induced by the correlations in the supercluster magnetic field. Within the framework of this model the authors extracted the spectral index of the magnetic field on supercluster scales of order 100 Mpc. The extracted value was found to be equal to 2.74 ± 0.04 . Cosmological magneto-hydrodynamic simulations (Dolag et al., 2002) on cluster scales of order few Mpc lead to a spectral index of 2.70 which is, surprisingly, in good agreement with the value extracted in Tiwari and Jain (2015b). However this may be merely a coincidence since the two refer to very different distance scales. The effect claimed in Tiwari and Jain (2015b) needs to be tested carefully by future surveys. The alignment might arise due to bias and furthermore it is found that the significance of the effect reduces considerably if the jackknife errors are taken into account (Tiwari and Jain, 2015b). The authors argued that we require at least four times larger data set in order to have a reliable confirmation of this effect.

2.7 Other Anomalies

Other CMB anomalies worth mentioning are the Cold Spot and the parity asymmetry. Cruz et al. (2005) reported an anomalous cold spot at $(l = 209^\circ, b = -57^\circ)$ with a size of 10° . To understand the parity asymmetry we have to think of the temperature field being sum of even and odd parity fields. The even and odd parity can be characterised by

$$P^+ = \sum_{\ell=2}^{\ell_{\max}} 2^{-1} (1 + (-1)^\ell) \ell(\ell+1) / 2\pi C_\ell \quad (21)$$

$$P^- = \sum_{\ell=2}^{\ell_{\max}} 2^{-1} (1 - (-1)^\ell) \ell(\ell+1) / 2\pi C_\ell \quad (22)$$

The ratio P^+/P^- denotes the ratio of the even parity contribution to the odd parity contribution. It was reported around 2010 (Kim and Naselsky, 2010; Aluri and Jain, 2012), that the

ratio is anomalously large when summing over the largest angular scales. Summing the multipoles $2 \leq \ell \leq 22$ the results for the ratio for WMAP 7 year data was 0.71, indicating a larger contribution from the even parity. Both these anomalies continue to exist in the Planck CMB data.

3 Theoretical Expectations

It is generally believed that the effects reviewed in the previous section are inconsistent with the Big Bang cosmological model. Although these observations appear to be in conflict with the cosmological principle, it has been shown in (Aluri and Jain, 2012; Rath et al., 2013) that they can be accommodated within the Big Bang paradigm. The basic idea is that the early pre-inflationary phase of the Universe may not be isotropic and homogeneous. It acquires this property during the early phase of inflation. This has been explicitly demonstrated for the case of Bianchi models (Wald, 1983) which are anisotropic but homogeneous. It has also been shown that, for a wide range of parameters, modes generated during this early period can re-enter the horizon before the current era and hence affect observations (Aluri and Jain, 2012; Rath et al., 2013). This implies that although the background evolution is isotropic and homogeneous, the perturbations need not respect the cosmological principle. Interestingly the dominant effect is expected for low k modes, which observationally appear to show the largest deviation from isotropy. This phenomenon has been explicitly demonstrated in Rath et al. (2013) where the quadrupole and octopole alignment is explained in terms of an early anisotropic phase of inflation. Similar ideas have been explored in order to explain the hemispherical anisotropy (Rath et al., 2015; Jain and Rath, 2015; Kothari et al., 2015a; Ghosh et al., 2016). However in this case an explicit model requires either an inhomogeneous Universe (Carroll et al., 2010; Rath et al., 2015) or space-time noncommutativity (Jain and Rath, 2015; Kothari et al., 2015b). A detailed analysis of such models is so far not available in the literature. Here we briefly review some basic results which have been obtained by assuming a model of primordial power spectrum.

Let us first consider the primordial power spectrum in real space, defined as,

$$F(\mathbf{R}, \mathbf{X}) = \langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle \quad (23)$$

where $\delta(\mathbf{x})$ is the primordial density fluctuation at comoving coordinate \mathbf{x} , $\mathbf{R} = \mathbf{x} - \mathbf{x}'$ and $\mathbf{X} = (\mathbf{x} + \mathbf{x}')/2$. In Kothari et al. (2015a) the authors consider the following inhomogeneous model:

$$F(\mathbf{R}, \mathbf{X}) = f_1(R) + \sin\left(\lambda \cdot \frac{\mathbf{X}}{\tau_0} + \delta\right) f_2(R), \quad (24)$$

where $\hat{\lambda}$ and δ are parameters and τ_0 is the current conformal time. Here the second term represents the contribution due to inhomogeneity. In Fourier space, the corresponding power spectrum is given by,

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = P_{\text{iso}}(k) \delta^3(\mathbf{k} - \mathbf{k}') - \frac{i}{2} g(k_+) \left[\delta^3\left(\mathbf{k} - \mathbf{k}' + \frac{\lambda}{\tau_0}\right) - \delta^3\left(\mathbf{k} - \mathbf{k}' - \frac{\lambda}{\tau_0}\right) \right] \quad (25)$$

where

$$g(k_+) = \int \frac{d^3 R}{(2\pi)^3} \exp\left[i(\mathbf{k} + \mathbf{k}') \cdot \frac{\mathbf{R}}{2}\right] f_2(R),$$

and $\mathbf{k}_+ = (\mathbf{k} + \mathbf{k}')/2$. This model leads to correlations between multipoles l and $l \pm 1$ of CMB, as expected in the case of dipole modulated temperature field (see Eq. 19). Kothari et al. (2015a) parameterize the function $g(k)$ as a power law, i.e.,

$$g(k) = g_0 P_{\text{iso}}(k) (k\tau_0)^{-\alpha} \quad (26)$$

where g_0 and α are parameters. A fit to the CMB dipole modulation data suggests that $\alpha \approx 1$. A similar analysis has also been carried out for an anisotropic but homogeneous model (Kothari et al., 2015a). As explained earlier, such a model is not possible in commutative spacetimes. However it may arise within the framework of noncommutative spacetimes.

A study of the implications of such a primordial model on large scale structures is so far not available in the literature. We expect that predictions based on such models will become available by the time SKA becomes operational.

3.1 The galaxy power spectrum

For tests at SKA our primary aim is to study the distribution of galaxies at large distances or equivalently their angular power spectrum C_l . We next briefly discuss the relation between Λ CDM power spectrum $P(k)$ to C_l . Let $\mathcal{N}(\hat{r})$ be the projected number density (per steradian) in the direction \hat{r} , and $\bar{\mathcal{N}}$ be the mean number density averaged over the sky. We write the number density $\mathcal{N}(\hat{r}) = \bar{\mathcal{N}}(1 + \Delta(\hat{r}))$, where $\Delta(\hat{r})$ represents the projected number surface density contrast. Let the three-dimensional dark matter density contrast be represented as $\delta_m(\mathbf{r}, z(r))$, where $(\mathbf{r}, z(r))$ represent a unique location in space and time. The vector \mathbf{r} stands for comoving distance r in direction \hat{r} and $z(r)$ is the redshift corresponding to comoving distance r . Assuming linear galaxy biasing $b(z)$ and linear growth factor $D(z)$ of density contrast we write the corresponding galaxy contrast $\delta_g(\mathbf{r}, z(r)) = \delta_m(\mathbf{r}, z = 0)D(z)b(z)$. Now we can write the theoretical expression for $\Delta(\hat{r})$ as,

$$\begin{aligned}\Delta(\hat{r}) &= \int_0^\infty \delta_g(\mathbf{r}, z(r))p(r)dr \\ &= \int_0^\infty \delta_m(\mathbf{r}, z = 0)D(z)b(z)p(r)dr,\end{aligned}\tag{27}$$

where $p(r)dr$ is the probability of observing a galaxy between r and $(r + dr)$. The expansion of $\Delta(\hat{r})$ in spherical harmonics and subsequent harmonic coefficients, \tilde{a}_{lm} , similar to equation (7), is given as,

$$\begin{aligned}\tilde{a}_{lm} &= \int d\Omega \Delta(\hat{r})Y_{lm}(\hat{r}) \\ &= \int d\Omega Y_{lm}(\hat{r}) \int_0^\infty \delta_m(\mathbf{r}, z = 0)D(z)b(z)p(r)dr.\end{aligned}\tag{28}$$

To write the harmonic coefficients, \tilde{a}_{lm} , in terms of the k -space density field $\delta_{\mathbf{k}}$, we expand $\delta_m(\mathbf{r}, z = 0)$ in Fourier domain,

$$\delta_m(\mathbf{r}, z = 0) = \frac{1}{(2\pi)^3} \int d^3k \delta_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}},\tag{29}$$

and substitute

$$e^{i\mathbf{k}\cdot\mathbf{r}} = 4\pi \sum_{l,m} i^l j_l(kr) Y_{lm}^*(\hat{r}) Y_{lm}(\hat{k}),$$

where j_l is the spherical Bessel function of first kind for integer l . Subsequently we write

$$\tilde{a}_{lm} = \frac{i^l}{2\pi^2} \int D(z)b(z)p(r)dr \int d^3k \delta_{\mathbf{k}} j_l(kr) Y_{lm}^*(\hat{k}).\tag{30}$$

Following equation (30) we write the theoretical angular power spectrum \tilde{C}_l as,

$$\begin{aligned}\tilde{C}_l &= \langle |\tilde{a}_{lm}|^2 \rangle \\ &= \frac{2}{\pi} \int dk k^2 P(k) \left| \int_0^\infty D(z)b(z)p(r)dr j_l(kr) \right|^2 \\ &= \frac{2}{\pi} \int dk k^2 P(k) W^2(k).\end{aligned}\tag{31}$$

where $W(k) = \int_0^\infty D(z)b(z)p(r)drj_l(kr)$ is the window function in k -space. We have also used $\langle \delta_{\mathbf{k}}\delta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}')P(k)$ where $P(k)$ is Λ CDM power spectrum.

3.2 Observational C_l

The observational estimate of C_l analogous to theoretical \tilde{C}_l given in equation (31) is,

$$C_l^{\text{obs}} = \frac{\langle |a'_{lm}|^2 \rangle}{J_{lm}} - \frac{1}{\mathcal{N}} \quad (32)$$

where $a'_{lm} = \int_{\text{survey}} d\Omega \Delta(\hat{r}) Y_{lm}(\hat{r})$ and $J_{lm} = \int_{\text{survey}} |Y_{lm}|^2 d\Omega$, the J_{lm} is an approximate correction factor for the partial survey region (Peebles, 1980). The term $\frac{1}{\mathcal{N}}$ removes the contribution from the Poissonian shot-noise.

The error in above estimate of power spectrum due to cosmic variance, sky coverage and shot-noise is as follows:

$$\Delta C_l = \sqrt{\frac{2}{(2l+1)f_{\text{sky}}}} \left(C_l^{\text{obs}} + \frac{1}{\mathcal{N}} \right) \quad (33)$$

where f_{sky} is the fraction of sky observed in the survey. Notice that the above error estimate is applicable in case of the 2-point galaxy-galaxy angular power spectrum (C_l^{gg}). The lensing shear power spectrum is deduced considering shape measurements of the galaxies. The shear angular power spectrum error estimate is given by,

$$\Delta C_l = \sqrt{\frac{2}{(2l+1)f_{\text{sky}}}} \left(C_l^{\text{obs}} + \frac{\sigma_\epsilon^2}{\mathcal{N}} \right) \quad (34)$$

where σ_ϵ is the RMS variance of the ellipticity distribution. Furthermore, for the case of polarized sources, assuming that the polarization position angle is an unbiased tracer of the intrinsic morphological orientation of the galaxy with a scatter of α_{rms} , the corresponding error estimate is as follows (Brown and Battye, 2011b,a):

$$\Delta C_l = \sqrt{\frac{2}{(2l+1)f_{\text{sky}}}} \left(C_l^{\text{obs}} + \frac{16\alpha_{\text{rms}}^2 \sigma_\epsilon^2}{\mathcal{N}} \right). \quad (35)$$

4 Tests of statistical isotropy at SKA

We propose the following tests of statistical isotropy in large scale structures:

1. Determination of the dipole in number counts and sky brightness of radio sources in order to test its consistency with the kinematic dipole.
2. Determination of the dipole in number counts of significantly polarized radio sources as well as in the polarized flux.
3. Testing the alignment of linear polarizations of radio sources as a function of their relative separation.
4. Testing the presence of dipole modulation in radio sources.
5. Determination of the dipole anisotropy in the offsets between linear polarization angles and the galaxy orientation angles.

4.1 SKA technical details and capabilities

The SKA will be a highly flexible instrument with unprecedented observational capabilities. It will consist of an inner core and outer stations arranged in a log-spiral pattern. The full array will be extended to at least 3000 km from the central core. This will be the largest radio telescope in the world and will revolutionize our understanding of the Universe. The SKA will operate in frequency range from 70 MHz to 10 GHz (see Dewdney et al. (2013) for more details).

The SKA will perform both redshift (HI) and radio continuum surveys in the aforementioned frequency range. There will be two phases of SKA observations. The final phase is expected to map out 1 billion galaxies over a sky area of $f_{\text{sky}} \sim 3/4$, out to a redshift of $z \sim 2$. This will reduce the shot-noise in galaxy angular power spectrum (see equation (32)) by a factor of 3000. The resulting shot-noise will be 3 orders of magnitude lower than $\Lambda\text{CDM } \tilde{C}_l$ and will be negligible in comparison to cosmic variance (equation (33)).

The SKA will yield measurements of various cosmological parameters with unmatched precision. The anisotropy tests at various scales will improve immensely. The dipole anisotropy observed in NVSS brightness and polarization will be clearly settled. At present the signal is observed at $\sim 3\sigma$ (Tiwari et al., 2015). The radio galaxy biasing consideration gives similar significance for reasonable radial number density and galaxy bias values (Tiwari and Nusser, 2015). The galaxy-bias is a nuisance in relating the galaxy clustering to underlying dark matter distribution. The biasing is almost stochastic, scale-dependent, redshift dependent and non-linear (Dekel and Lahav, 1999). The bias determination is almost always indirect as we always need the underlying dark matter density power spectrum to extract bias from galaxy clustering. As discussed earlier, the NVSS total source count is $\sim 1.8 \times 10^6$. The SKA source count is expected to be roughly two orders of magnitude larger (Wilman et al., 2008). This also applies to the polarized source density. The wide and deep polarization surveys with SKA will reach to μJy flux limit. The deep polarization survey ($2 \mu\text{Jy}$) will probe the source population as a function of flux, luminosity and redshift, whereas the wide ($33,000 \text{ deg}^2$, sensitive up to $10 \mu\text{Jy}$) survey will reveal the large scale clustering of polarized galaxies. Hence the statistical error in source counts, sky brightness, polarized number count as well as polarized flux will be sufficiently small in order to reliably extract the signal of dipole anisotropy. However one has to carefully remove systematic effects from data.

Besides the galaxy biasing described above, the most important systematic effect is the contribution due to local clustering dipole (Blake and Wall, 2002; Singal, 2011; Gibelyou and Huterer, 2012; Rubart and Schwarz, 2013; Tiwari et al., 2015; Schwarz et al., 2015). So far this has been removed by cross correlating with catalogues of known nearby galaxies (Blake and Wall, 2002). With SKA redshift survey the exact radial number density will be known. The large area survey coverage and depth in redshift with SKA observation will allow us to measure the galaxy clustering at the largest scale ever. The SKA galaxy power spectrum will cover the turnover ($k < 0.02 \text{ h Mpc}^{-1}$) of ΛCDM power spectrum. This will also allow a better constrain on galaxy bias. The NVSS survey also suffers from significant declination bias due to two different array configurations used for different declinations. While this may not be an issue for SKA, a declination bias centered at the array location may arise (Tiwari and Jain, 2015a). Such a bias has been identified in the NVSS survey, particularly for the sample with low flux cutoff, and can be effectively removed by the procedure described in (Tiwari and Jain, 2015a). Yet another systematic effect arises in relating the extracted dipole from data to the local speed. The main issue here is the deviation of the distribution of number density $n(S)$ as a function of the flux S from a pure power law. However it has been shown that a generalized distribution fits the data very well and one can extract the local speed very accurately using this fit (Tiwari et al., 2015; Tiwari and Jain, 2015a).

Further the resolved shape of billion galaxies from SKA will give the best shear measurements. The light rays from distant galaxies follow the geodesics, which bend according to the presence of matter in intervening space. This results in a shape distortion following the matter

distribution fluctuations along the line of sight. This enables a direct mapping of mass distribution (luminous + non-luminous) and dark energy measurements. The statistical error in auto-shear power spectrum with SKA will decrease by a factor of ~ 3000 due to high number surface density ($\sim 10^5 \text{ deg}^2$) and reliable shape measurements (Demetroullas and Brown, 2016). With such huge improvement in statistics, it will be challenging to control the corresponding systematics. Cross-correlations between shear maps from SKA and LSST/*Euclid* can remove observational systematics.

The enhanced polarization survey at SKA will also allow us to reliably test the alignment of linear polarizations as a function of the angular separation among galaxies (Tiwari and Jain, 2013, 2015b). With two orders of magnitude increase in the number of sources, the effect will be seen clearly if present in data. Furthermore the SKA redshift survey would allow a 3 dimensional analysis which will provide an unambiguous test of this phenomenon, both at the supercluster scale (Tiwari and Jain, 2013, 2015b) and on larger cosmological distance scales (Pelgrims and Hutsemékers, 2015). Within the framework of the theoretical model of Tiwari and Jain (2015b), it will allow a clean extraction of the spectral index of the supercluster magnetic field at distance scales of order 100 Mpc. On cluster scales of order few Mpc, cosmological magneto-hydrodynamic simulations lead to a spectral index of 2.7 for the corresponding magnetic field. It may be interesting to apply the formalism proposed in Tiwari and Jain (2015b) and extract the magnetic field spectral index by studying correlations between the radio linear polarizations at this distance scale. This will require large amount of data on linear polarizations of galaxies separated by distances of order Mpc. Such a measurement may also be feasible at SKA.

SKA will also make measurements of linear polarizations at different frequencies for a very large sample of sources (Beck and Gaensler, 2004; Haverkorn et al., 2015). The main purpose of these observations is the determination of Faraday rotation measures which will provide information about the milky way magnetic field. However these will also allow measurements of the host polarization position angles. For the case of active galaxies, if we are also able to determine the orientation of the jets, it is possible to test the dipole anisotropy claimed in Jain and Ralston (1999). We point out that extraction of rotation measures and polarization position angles may be facilitated by the refined technique developed in Sarala and Jain (2002).

5 Discussion and Conclusions

The tantalizing possibility that the Cosmological principle may be violated is indicated by many observations. The most prominent of these effects is the so called Virgo Alignment, which refers to a wide range of phenomena indicating a preferred direction pointing towards Virgo. The SKA has the capability to convincingly test several of these effects. These include the dipole anisotropy in radio polarization angles (Jain and Ralston, 1999), the dipole in the number counts and sky brightness (Blake and Wall, 2002; Singal, 2011; Gibelyou and Huterer, 2012; Tiwari et al., 2015; Rubart and Schwarz, 2013) and in the polarized number counts and polarized flux (Tiwari and Jain, 2015a). These observations may indicate that we need to go beyond the standard Big Bang cosmology. Alternatively they may be explained by pre-inflationary anisotropic and/or inhomogeneous modes (Aluri and Jain, 2012; Rath et al., 2013). In either case, confirmation of this alignment effect is likely to revolutionize cosmology. SKA will also test the signal of dipole modulation in large scale structure. Finally it will test the alignment of radio polarizations. It has been suggested that the alignment is induced by the correlations in the cluster magnetic field (Tiwari and Jain, 2015b). Hence, if confirmed, this phenomenon might provide a tool to study the statistical properties of the large scale magnetic field.

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